## Min-Max & Other Comparisons

You have a rectangular array of numbers. For each row, note the largest in that row, and for each column, note the smallest in that column. What if anything can be said about the smallest of the largest in each row, and the largest of the smallest in each column?

Same array but this time you calculate the arithmetic mean of each row, and the geometric mean of those numbers. You also calculate the geometric mean of the columns, and the arithmetic mean of those numbers. Which answer is the larger?

## Tails of the Unexpected

In front of you there are a number of coins on a table, but you cannot seem them or touch them. You are told how many are showing tails. Give instructions to someone (involving choosing some coins without regard to which way up they are, and turning over a specified number of coins0 so that there will be two piles of coins showing the same number of tails.

## Seas & Islands

The following diagrams show that recording the number of seas and the number of islands does not determine the configuration uniquely. Given a configuration with *s* seas and *i* islands, find a way to construct a configuration with *i* seas and *s* islands.

[1, 2]

[2, 2]

[2, 2]

How many different configurations can have *s* seas and *i* islands?

What other information could be used to characterise a configuration?

## Quantifier Duels

Write down a statement which uses two or more quantifiers, such as

For all *a* there exists *b* such that b > *a*

together with the statement made by interchanging ‘for all’ and ‘there exists’:

There exits *a* such that for all *b*, *b* > *a*

Are either of them true? Can the arithmetic relation be adjusted to make both true?

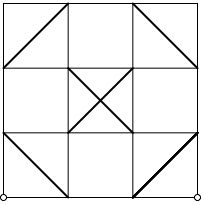
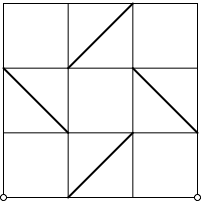
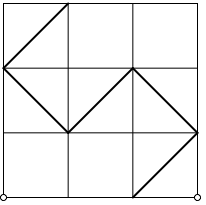
Another example:

For all *a* there exists *b* < *a* such that for all *c* > *a*, *c* – *b* > 1.

As a two person game, on their turn, each player adds a quantifier and an arithmetic condition or relationship involving all of the previous variables which is nevertheless true. The ‘game’ ends when the statement can no longer be extended truthfully.

## Grid Duals

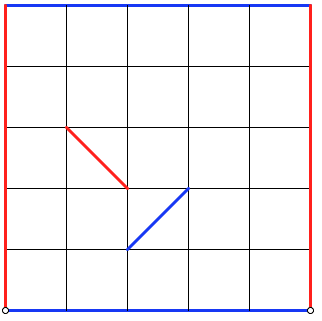
On a square grid of cells, insert diagonals in some cells so that if all the diagonals are interchanged, the grid remains invariant under some rotation. For example:

How many essentially different ones can you find of a given size? (See applet *Grids*)

## Diagonal paths

On an *n* by *n* square grid of cells, each player can insert a diagonal of their own colour in any cell that does not yet have one. The first to get a line joining their assigned opposite sides of the grid wins.



See applet *Diagonal Paths*